

Calc 213, sections 301 & 302

Quizzes for week 8, spring 2008

Monday's Quiz

Evaluate:

$$\int_1^{\infty} \frac{\ln(x)}{x^2} dx$$

Solution

We integrate from 1 to b , and then let b go to ∞ .

$$\int_1^{\infty} \frac{\ln(x)}{x^2} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{\ln(x)}{x^2} dx$$

We will evaluate this integral, and leave the limit until the last step. Next, we integrate by parts

$$\begin{aligned} u &= \ln(x) \text{ and } dv = x^{-2} dx \\ du &= x^{-1} dx \text{ and } v = -x^{-1} \end{aligned}$$

$$\begin{aligned} \int_1^b \frac{\ln(x)}{x^2} dx &= -\frac{\ln(x)}{x} \Big|_1^b + \int_1^b x^{-2} dx \\ &= \left[-\frac{\ln(b)}{b} - 0 \right] - x^{-1} \Big|_1^b \\ &= \left[-\frac{\ln(b)}{b} - 0 \right] - \frac{1}{b} + 1 \end{aligned}$$

Now we take the limit:

$$\begin{aligned} \int_1^{\infty} \frac{\ln(x)}{x^2} dx &= \lim_{b \rightarrow \infty} -\frac{\ln(b)}{b} - \frac{1}{b} + 1 \\ &= -0 - 0 + 1 \\ &= 1 \end{aligned}$$

Wednesday's Quiz

Evaluate:

$$\int_1^{\infty} \frac{\ln(x)}{x^{3/2}} dx$$

Solution

We integrate from 1 to b , and then let b go to ∞ .

$$\int_1^{\infty} \frac{\ln(x)}{x^{3/2}} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{\ln(x)}{x^{3/2}} dx$$

We will evaluate this integral, and leave the limit until the last step. Next, we integrate by parts

$$\begin{aligned} u &= \ln(x) \text{ and } dv = x^{-3/2} dx \\ du &= x^{-1} dx \text{ and } v = -2x^{-1/2} \end{aligned}$$

$$\begin{aligned}\int_1^b \frac{\ln(x)}{x^{3/2}} dx &= -2 \frac{\ln(x)}{x^{1/2}} \Big|_1^b + 2 \int_1^b x^{-3/2} dx \\ &= \left[-2 \frac{\ln(b)}{b^{1/2}} - 0 \right] - 4x^{-1/2} \Big|_1^b \\ &= \left[-2 \frac{\ln(b)}{b^{1/2}} - 0 \right] - 4 \frac{1}{b^{1/2}} + 4\end{aligned}$$

Now we take the limit:

$$\begin{aligned}\int_1^\infty \frac{\ln(x)}{x^{3/2}} dx &= \lim_{b \rightarrow \infty} -2 \frac{\ln(b)}{b^{1/2}} - 4 \frac{1}{b^{1/2}} + 4 \\ &= -0 - 0 + 4 \\ &= 4\end{aligned}$$