Calc 213, sections 301 & 302

Quiz for week 11, spring 2008

Suppose one of you company's divisions hires both college grads, and high school grads. Employees with a high school diploma get a salary of $\$30K$, while employees with a Bachelor's degree receive $\$60K$ annually. This division hires a mix of both kinds of employees with a total of x college grads, and y high school graduates. Their total budget for salaried employees is $\$600K$ (and they use up the entire budget). The production as a function of labour is $f(x,y) = x(60-x)+y(30-y)-10xy$. How many of each kind of employee should they hire?

i.e. Maximize the function

$$f(x,y) = x(60-x) + y(30-y) - 10xy$$

subject to the constraint

$$60x + 30y = 600$$

Solution

There's really 2 ways you can do this. You could solve the constraint for one of the variables (e.g. $y = 20-2$) and then plug that into $f(x,y)$ to get a function of a single variable which you can then maximize in the usual way. Or you could use the Lagrange multiplier method - this second approach is the one used here.

First we rewrite the constraint: $g(x,y) = 60x + 30y - 600 = 0$, then we set up the Lagrangian;

$$\begin{aligned} F(x,y,\lambda) &= f - \lambda g \\ &= x(60-x) + y(30-y) - 10xy - \lambda(60x + 30y - 600) \end{aligned}$$

At a maximum point, all derivatives must be zero;

$$\begin{aligned} \partial_x F = &\quad 60 - 2x - 10y - 60\lambda &= 0 \quad (1)\\ \partial_y F = &\quad 30 - 2y - 10x - 30\lambda &= 0 \quad (2)\\ \partial_\lambda F = &\quad 60x + 30y - 600 &= 0 \quad (3) \end{aligned}$$

We'll solve (1) and (2) for 30λ, and thereby eliminate λ

Divide (1) by 2 and rearrange to get;

$$30 - x - 5y = 30\lambda$$

from (2) we have;

$$30 - 2y - 10x = 30\lambda$$

Putting these 2 together we get;

$$\begin{aligned} & 30 - 2y - 10x = 30 - x - 5y \\ \Rightarrow \quad & 5y - 2y = 10x - x \\ \Rightarrow \quad & 3y = 9x \\ \Rightarrow \quad & y = 3x \end{aligned}$$

Then plug this into the constraint equation (3);

$$\begin{array}{rrcl} & 60x + 30(3x) - 600 & = & 0 \\ \Rightarrow & 150x & = & 600 \\ \Rightarrow & x & = & 4 \end{array}$$

Since $y = 3x$, we have $y = 12$. So your constrained maximum will occur at $x = 4, y = 12$.