Calc 213, sections 301 & 302

Quiz for week 14, spring 2008

Recall that

$$\sum_{k=0}^{n-1} ar^k = a\frac{1-r^n}{1-r}$$

Monday's Quiz

Simplify

$$\sum_{k=4}^{11} 7(\frac{1}{2})^k$$

Solution

There's 2 ways you can do this; either by re-indexing (so that $k = 4$ corresponds to $j = 0$ and $k = 11$ corresponds to $j = 7$), or by writing the sum as the difference of two other sums.

Method 1:

$$\sum_{k=4}^{11} 7(\frac{1}{2})^k = 7(1/2)^4 + 7(1/2)^5 + \ldots + 7(1/2)^{11}$$

factor out the first term (this is your new "a")

$$\begin{aligned} &= 7(1/2)^4\big(1 + 1/2 + \ldots + (1/2)^7\big) \\ &= \sum_{k=0}^{7} \frac{7}{2^4}(\frac{1}{2})^k \\ &= \frac{7}{2^4}\frac{1-(1/2)^8}{1-1/2} \\ &= \frac{7}{2^3}\big(1-(1/2)^8\big) \end{aligned}$$

Method 2:

$$\begin{aligned} \sum_{k=4}^{11} 7(\frac{1}{2})^k &= \sum_{k=0}^{11} 7(\frac{1}{2})^k - \sum_{k=0}^{3} 7(\frac{1}{2})^k \\ &= 7\frac{1-(1/2)^{12}}{1-1/2} - 7\frac{1-(1/2)^4}{1-1/2} \\ &= 2\cdot 7\big(1-(1/2)^{12} - 1 + (1/2)^4\big) \\ &= 2\cdot 7\big(-(1/2)^{12} + (1/2)^4\big) \\ &= \frac{7}{2^3}\big(1-(1/2)^8\big) \end{aligned}$$

Wednesday's Quiz

Simplify

$$\sum_{k=3}^{11} 3(\frac{1}{2})^k$$

Solution

Same as above;

$$\sum_{k=3}^{11} 3(\frac{1}{2})^k = \frac{3}{2^2}\big(1 - (1/2)^9\big)$$